B.Tech. DEGREE EXAMINATION, JANUARY 2023
Thrid Semester
Electrical and Electronics Engineering
Theory: Electromagnetic Theory
(2013-14 Regulations)

ANSWER KEY
PART A (2X10 = 20 Marks)

1. State Divergent theorem (T1)

The divergence theorem is a mathematical statement of the physical fact that, **in the absence of the creation or destruction of matter, the density within a region of space can change only by having it flow into or away from the region through its boundary**.

$$\iint_S \vec{F} \cdot \hat{n}\, ds = \iiint_V \vec{\nabla} \cdot \vec{F}\, dv$$

2. Define Energy density (T1)

Its denoted as $u_E = \frac{1}{2}\varepsilon_0 E^2$ as the energy density, i.e. the energy per unit volume, in the electric field. The energy stored between the plates of the capacitor equals the energy per unit volume stored in the electric field times the volume between the plates.

3. List the properties of conductor & dielectric materials (T2)

These are the most important properties of dielectric materials.

1. Electric susceptibility
2. Dielectric polarization
3. Electric dipole moment
4. Electronic polarization

4. Write Poisson's & Laplace's equation (T1)

The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

The electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes Laplace's equation

$$\nabla^2 V = 0$$

5. State Biot Savarts law (model)

Biot savart law states that " magnetic field due to a current-carrying conductor at a distance point is inversely proportional to the square of the distance between the conductor and point, and the magnetic field is directly proportional to the length of the conductor, current flowing in the conductor"

Vector notation

$$dB \propto \frac{I\, dl \times r}{r^3}$$

6. Infer the Lorentz force equation for a moving charge

Lorentz force, the force exerted on a charged particle q moving with velocity v through an electric field E and magnetic field B. The entire electromagnetic force F on the charged particle is called the Lorentz force and is given by **F = qE + qv × B**.

7. Find the energy stored in inductor having current of 3A flowing through the inductor of 100mH

Given: L = 100mH; I = 3A W = ½ LI2. W = **450X10-3 Joules**.

8. Identify the relationship between magnetic fled intensity & magnetization

The magnetic intensity defines the forces that the poles of a magnet experiences in a magnetic field whereas the intensity of magnetization explains the change in the magnetic moment of a magnet per unit volume whereas , Induced magnetization is a process where you can magnetize a non-magnetic material. You can do so when you bring it under the influence of an external magnetic field.

9. Write the significance of displacement current

Displacement currents play a central role in the propagation of electromagnetic radiation, such as light and radio waves, through empty space. A traveling, varying magnetic field is everywhere associated with a periodically changing electric field that may be conceived in terms of a displacement current.

In electromagnetism, displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of electric displacement field. Displacement current has the same units as electric current, and it is a source of the magnetic field just as actual current is.

10. Express the values of skin depth for a plane wave propagation though the dielectric with attenuation constant of 0.2887Np/m (Nepers per meter)

CONCEPT:

- The skin effect is the tendency of an **alternating electric current to become distributed within a conductor** such that the current density is largest near the surface of the conductor and decreases with greater depths in the conductor.
- The electric current flows mainly at the skin of the conductor, between the outer surface and a level called the **skin depth**.
- Skin depth is defined as the **reciprocal of attenuation constant** i.e.,

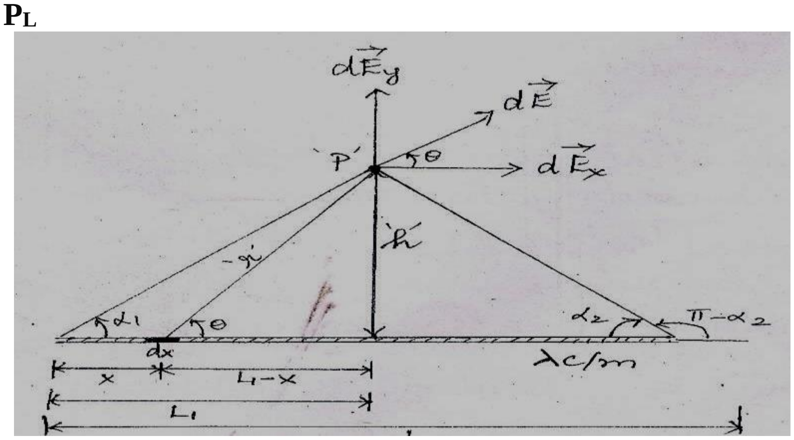
$$\delta = \frac{1}{\alpha}$$

= 1/0.2887 = 3.46m

PART B (5X11 = 55 Marks)

UNIT-I

11. Obtain the formula for the electric field intensity of an infinite long straight line carrying uniform line charge density of P_L



Assumptions:

Let the point 'p' be at a distance of 'L_1' meter from one end of the charged wire along 'x' axis, and at a height 'h' meter along 'y' axis.

Consider a small element 'dx' at a distance 'x' from one end of wire.

Charge element dq=λdx

Total charge 'q' due to full length will be the integration of dq.

Let 'dE' be the field due to charge element λdx

$$\vec{dE} = \vec{dE}\,\mathbf{x} + \vec{dE}\,\mathbf{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

From figure;

$$\vec{dE}_{x} = dE\cos\theta\, \vec{a}_x$$

$$\vec{dEy} = dE\,\sin\theta\, \vec{a}_y$$

By definition;

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}\,\vec{a}_r \quad \text{v/m}$$

$$\Rightarrow \vec{dE}_{x} = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$\vec{E}_{x} = \int_{x=0}^{L} \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta\,\vec{a}_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From the fig, $\tan\theta = \dfrac{h}{L_1 - x} \Rightarrow \cot\theta\, \dfrac{L_1 - x}{h}$

$\Rightarrow L_1 - x = h\cot\theta$

$\Rightarrow -dx = -h\,\text{cosec}^2\theta\, d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$

Also, $\sin\theta = \dfrac{h}{r} \Rightarrow r = h\,\text{cosec}\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$

Sub 3 and 4 in equation 2

$$\vec{Ex} = \int_{0}^{L} \frac{\lambda(h\cos ec^2\theta.d\theta)}{4\pi\varepsilon_0(h\cos ec\theta)^2}.\cos\theta\,\vec{a}_x$$

NOTE: [dx→become dθ,so change limits also ie, x=0 to L, becomes θ=α_1, to π – α_2]

$$\vec{E}_x = \int_{\alpha_1}^{\Pi-\alpha_2} \frac{\lambda}{4\pi\varepsilon_0 h}.\cos\theta\, d\theta$$

$$\Rightarrow \vec{Ex} = \frac{\lambda}{4\pi\varepsilon_0 h}[\sin\theta\,]_{\alpha_1}^{\Pi-\alpha_2} = \frac{\lambda}{4\pi\varepsilon_0 h}\,[\sin(\pi-\alpha_2) - \sin\alpha_1]$$

$$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 h}[\sin\alpha_2 - \sin\alpha_1] \quad \text{V/m}$$

Similarly

$$\mathbf{d}\,\vec{Ey} = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\ \sin\theta\, \vec{ay} \qquad \Rightarrow \vec{Ey} = \int_{x=0}^{L} \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\ \sin\theta$$

Substituting for 'θ' and 'dx'

$$\vec{Ey} = \int_{\alpha_1}^{\Pi=\alpha_2} \frac{\lambda h \cos ec^2\theta d\theta}{4\pi\varepsilon_0 (h^2 \cos ec^2\theta)} \sin\theta\, \vec{ay} = \int_{\alpha_1}^{\Pi=\alpha_2} \frac{\lambda}{4\pi\varepsilon_0 h}\ \sin\theta\ d\theta\, \vec{ay} = \frac{\lambda}{4\pi\varepsilon_0 h}\left[1-\cos\theta\right]_{\alpha_1}^{\Pi-\alpha_2} \vec{a_y}$$

$$\Rightarrow \vec{Ey} = \frac{\lambda}{4\pi\varepsilon_0 h}[\cos\alpha_1 + \cos\alpha_2]\ \vec{a_y}\ \text{V/m}$$

Case (i)

If point 'p' is along $\perp$ **r bisector of wire, than** $\alpha_1 = \alpha_2 = \alpha \therefore \vec{a_x}$ **component get cancelled and** $\vec{a_y}$ **component adds.**

$$\vec{E_y} = \vec{E} = \frac{\lambda}{4\pi\varepsilon_0 h}[\cos\alpha + \cos\alpha]$$

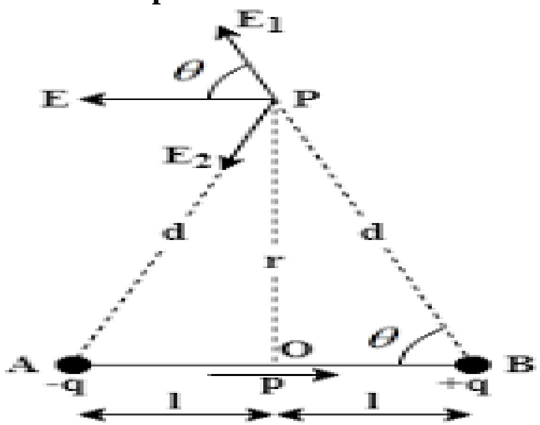
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 h}.\cos\alpha\ \vec{a_y}\ \text{V/m}$$

Case (ii)

In case of infinitely long conductor, $\alpha = 0$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 h}\vec{a_y}\ \ \text{v/m}$$

12. Derive the expression for potential due to an electric dipole at any point P also find the electric fields intensity and in terms of dipole moment



Let an electric dipole consist of two equal and opposite point charges $-q$ at A and $+q$ at B ,separated by a small distance AB =2a ,with centre at O.

The dipole moment, $p = q \times 2a$

We will calculate potential at any point P, where

$OP = r$ and $\angle BOP = \theta$

Let $BP = r_1$ and $AP = r_2$

Draw AC perpendicular PQ and BD perpendicular PO

In ΔAOC $\cos\theta = OC/OA = OC/a$

$OC = a\cos\theta$

Similarly, $OD = a\cos\theta$

Potential at P due to $+q = \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$

And Potential at P due to $-q = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$

Net potential at P due to the dipole

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_2} - \frac{q}{r_1}\right)$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Now, $r_1 = AP = CP$

$= OP + OC$

$= r + a\cos\theta$

And $r_2 = BP = DP$

$= OP - OD$

$= r - a\cos\theta$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

$$= \frac{p\cos\theta}{r^2 - a^2\cos^2\theta} \text{(Since } p = 2aq)$$

Special cases:-

(i) When the point P lies on the axial line of the dipole, $\theta = 0°$

$\cos\theta = 1$

$$V = \frac{p}{r^2 - a^2}$$

If $a << r$, $V = \frac{p}{r^2}$

Thus due to an electric dipole ,potential, $V \propto \frac{1}{r^2}$

(ii) When the point P lies on the equatorial line of the dipole, $\theta = 90°$

$\cos\theta = 0$

i.e electric potential due to an electric dipole is zero at every point on the equatorial line of the dipole.

UNIT II

13. Find the value of capacitance of a capacitor consisting of two parallel metal plates of 30cm*30cm surface area separately by 5 mm in air. What is the total energy stored by capacitor is charged to a potential difference of 1000 V . What is energy density?

Sol :- Area of the parallel plate capacitor = $30 \times 30\ cm^2 = 900 cm^2$

Distance separating of plates = $d = 5\ mm = 5 \times 10^{-3} m$

potential different across the plate = $V = 1000$ volts

The formula for capacitance will be

$$C = \varepsilon_0 A/d$$

electrostatic energy stored in capacitor is given by

$$E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{d} \cdot V^2$$

ε_0 = permittivity of free space = $8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

$$E = \frac{1}{2} \times \frac{8.854 \times 10^{-12} \times 900 \times 10^{-4}}{5 \times 10^{-3}} \times 1000^2$$

$$= 7.9687 \times 10^{-12} \times 10^{-4} \times 10^{+3} \times 10^{6} \times 10^{2}$$

$$= 7.9687 \times 10^{-16} \times 10^{11} = 7.9687 \times 10^{-5} J$$

∴ The stored electrostatic energy inside the capacitor is $7.9687 \times 10^{-5} J$

Energy density $U = \frac{E_1}{V} = \frac{E_1}{A \times d} = \frac{7.9687 \times 10^{-5}}{900 \times 10^{-3} \times 5 \times 10^{-3}}$

$$= \frac{7.9687 \times 10^{-5}}{4500 \times 10^{-6}} = 0.00177 \times 10^{-5} \times 10^{6}$$

$$= 0.0177 J/m^3$$

14. Solve 1D Laplace eq to obtain the fields inside a parallel plate capacitor & also evaluate the expression for the surface charge density at two plates

Electric field on the inside of the parallel plate capacitor due to one of the plates is given by:

$$E_1 = \frac{\sigma}{2\varepsilon_o} = \frac{Q}{2A\varepsilon_o}$$

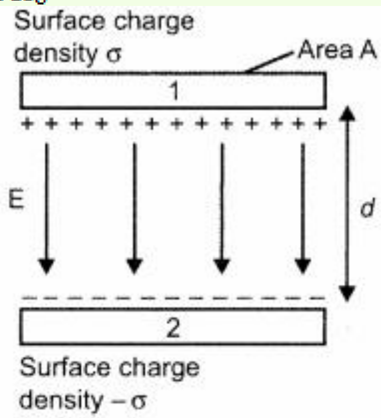
Total electric field due to the two plates is given by:

$$E = 2E_1 = \frac{Q}{A\varepsilon_o}$$

Potential difference between the plates is given by:

$$V = Ed$$

$$V = \frac{Qd}{A\varepsilon_o}$$



A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance. The two plates have charges q and $-q$ and distance between them is d.

Plate 1 has charge density, $\sigma = \frac{q}{A}$

Plate 2 has charge density, $\sigma = -\frac{q}{A}$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up.

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

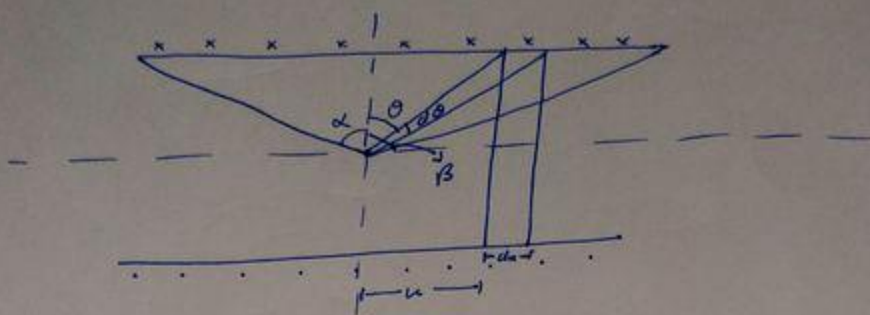
UNIT III

15. State & explain the Ampere's circuit law & show that the field strength at the end of a solenoid is one half of that at the center

General Solenoid

R, l

$n \rightarrow$ no. of turns per unit length



$$x = R\tan\theta \qquad \frac{dx}{d\theta} = R\sec^2\theta$$

$$dB = \frac{\mu_0 n I R^2}{2(x^2+R^2)^{3/2}}\,dx \qquad B = \int dB = \frac{\mu_0 n I R^2}{2}\int \frac{dx}{(x^2+R^2)^{3/2}}$$

$$B = \frac{\mu_0 n I R^2 \cdot R}{2R^2}\int \frac{\sec^2\theta\, d\theta}{\sec^3\theta} = \frac{\mu_0 n I}{2}\int_{-\alpha}^{\beta} \cos\theta\, d\theta$$

$$B = \frac{\mu_0 n I}{2}(\sin\beta + \sin\alpha)$$

$\therefore$ B at middle for Ideal Solenoid $= \frac{\mu_0 n I}{\cancel{2}} \cdot \cancel{2}\sin\alpha = \mu_0 n I$

$\therefore$ B at one end for Ideal Solenoid $= \frac{\mu_0 n I}{2} \cdot \sin\alpha = \frac{\mu_0 n I}{2}$

16. Derive magnetic scalar potential & vector potential with necessary eq.

MAGNETIC SCALAR AND VECTOR POTENTIALS

We recall that some electrostatic field problems were simplified by relating the electric Potential V to the electric field intensity **E** (**E** = -del V). Similarly, we can define a potential associated with magnetostatic field B. In fact, the magnetic potential could be scalar Vm vector A. To define Vm and A involves two important identities:

Del*(delV) = 0

Del.(del x A) = 0

which must always hold for any scalar field V and vector field A.

Just as **E = -**del**V**, we define the *magnetic scalar potential Vm* (in amperes) as related to **H** according to

H = **-** del **Vm** if **J** = 0

J =del x **H** = -del x (-del Vm) = 0

since Vm, must satisfy the condition in eq. Thus the magnetic scalar potential Vm is only defined in a region where J = 0 as in eq. We should also note that Vm satisfies Laplace's equation just as V does for electrostatic fields; hence,

del ^{2}Vm = 0, (**J** = 0)

SCALAR MAGNETIC POTENTIAL

Like scalar electrostatic potential, it is possible to have scalar magnetic potential. It is defined in such a way that its negative gradient gives the magnetic field, that is,

H = del Vm

Taking curl on both sides, we get

del x **H** = -del x del Vm

But curl of the gradient of any scalar is always zero.

So, del x **H** =0

But, by Ampere's circuit law del* **H** = **J**

or, J = 0

In other words, scalar magnetic potential exists in a region where J = 0.

H = -del Vm (J=0)

The scalar potential satisfies Laplace's equation, that is, we have

del.B = µ0 del.H = 0 = mdel (-delVm) = 0

or,

del ^{2}Vm = 0 **(J = 0)**

Characteristics of Scalar Magnetic Potential (Vm)

1. The negative gradient of Vm gives H, or H = -□Vm
2. It exists where J = 0
3. It satisfies Laplac's equation.
4. It is directly defined as $$V_m = -\int_A^B H \cdot dL$$
5. It has the unit of Ampere.

VECTOR MAGNETIC POTENTIAL

Vector magnetic potential exists in regions where J is present. It is defined in such a way that its curl gives the magnetic flux density, that is,

B = del x **A**

where **A** = vector magnetic potential (wb/m).

It is also defined as

$$A \equiv \oint \frac{\mu_0 IdL}{4\pi R}\left(\frac{Henry - Amp}{m}\right)$$

or, $$A \equiv \oint_s \frac{\mu_0 Kds}{4\pi R},$$ (K = current sheet)

or, $$A \equiv \oint_v \frac{\mu_0 Jdv}{4\pi R},$$

Characteristics of Vector Magnetic Potential

1. It exists even when J is present.
2. It is defined in two ways

   $B \equiv \nabla \times A$ and

   $$\int_v \frac{\mu_0 Jd\,v}{4\pi R}$$

3. $\nabla^2 \mathbf{A} = \mu_0 j$
4. $\nabla^2 \mathbf{A} = 0$ if $J = 0$

UNIT IV

17. A solenoid is 50 cm long 2 cm in dia & 1500 turns. The cylindrical core has a diameter of 2 cm & relative permeability of 75. This is co-axial with 2nd which is 50 cm long 3 cm dia 1200 turns. Solve the inductance L for inner & outer solenoid.

Given

length of the solenoid, $l_1 = 50cm = 0.5m$

diameter, $d_1 = 2cm \Rightarrow R_1 = 0.01m$

Number of turns, $N_1 = 1500$

cylinder core diameter, $d_c = 2cm \Rightarrow R_c = 0.01m$

relative permeability, $\mu_r = 75$

$l_2 = 50cm = 0.5m$, $d_2 = 3cm \Rightarrow 1.5cm = R_2$, $N_2 = 1200$

$L_1 = ?$ and $L_2 = ?$

$$L_1 = \frac{\mu_1 N_1^2 S_1}{l} = \frac{\mu_r \mu_0 N_1^2 \pi D_1^2}{4l}$$

$$= \frac{(75)(4\pi \times 10^{-7}\,\mathrm{H/m})(1500)^2 \pi (.02\mathrm{m})^2}{4(.50\mathrm{m})}$$

$$L_1 = .133\ \mathrm{H} = 133\ \mathrm{mH}$$

Similarly

$$L2 = \frac{\mu_1 N_1^{\,2} S_1}{l} = \frac{\mu_r \mu_0 N_1^{\,2} \pi D_1^{\,2}}{4l}$$

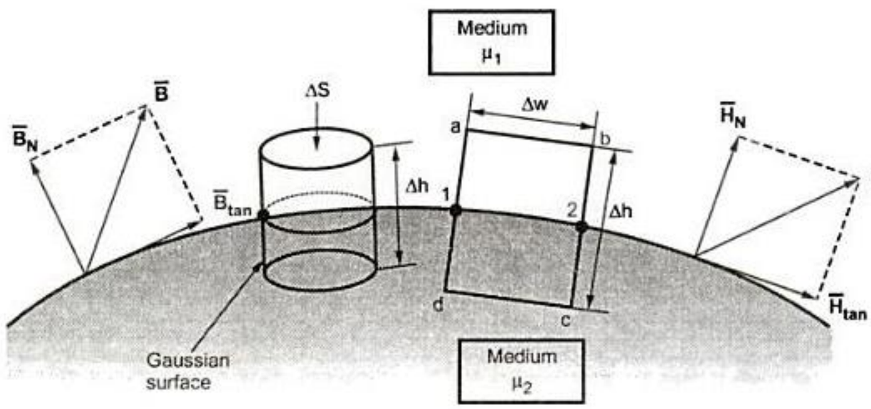
$$= \frac{(75)(4\pi \times 10^{-7}\,\mathrm{H/m})(1200)^2 \pi (.03\mathrm{m})^2}{4(.50\mathrm{m})}$$

$$L2 = 192\ \mathrm{mH}$$

18. Describe about the magnetic boundary condition at the interface between tow magnetic medium & derive the necessary boundary condition

The conditions of magnetic field existing at the boundary of the two media when the magnetic field passes from one medium to other are called boundary conditions or magnetic boundary conditionsIn the magnetic boundary condition, the condition of B and H are studied at the boundary B and H vectors are resolved into two components

1. **Tangential to boundary**
2. **Normal (perpendicular) to boundary**



Consider a boundary between two isotropic homogeneous linear materials with different permeabilitiesμ1 and μ2

Boundary conditions for normal component

To find the normal component of E select a closed Gaussian surface in the form of a right circular cylinder

Let the height be Δh and be placed in such a way the Δh / 2 is in medium1 and remaining Δh / 2 is in medium 2.

Also zxis of the cylinder is in the normal direction to the surface

According to gauss's law for the magnetic field

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

$$\oint_{top} \bar{B} \cdot d\bar{S} + \oint_{bottom} \bar{B} \cdot d\bar{S} + \oint_{lateral} \bar{B} \cdot d\bar{S} = 0$$

For top surfaces :

$$\oint_{Top} \bar{B} \cdot d\bar{S} = B_{N_1} \oint_{Top} d\bar{S} = B_{N_1} \Delta S$$

For bottom surface :

$$\oint_{Bottom} \bar{B} \cdot d\bar{S} = B_{N_2} \oint_{Bottom} d\bar{S} = B_{N_2} \Delta S$$

For lateral surface

$$\oint_{Lateral} \bar{B} \cdot d\bar{S} = 0$$

Sub the above equations

$$B_{N_1} \Delta S - B_{N_2} \Delta S = 0$$

BN1 and BN2 are in opposite direction

$$B_{N_1} \Delta S = B_{N_2} \Delta S$$

$$\boxed{B_{N_1} = B_{N_2}}$$

Thus the normal component of B is continuous at the boundary As the magnetic flux density and the magnetic field intensity are related by

$$\bar{B} = \mu \bar{H}$$

The above equation becomes

$$\mu_1 H_{N_1} = \mu_2 H_{N_2}$$

$$\boxed{\frac{H_{N_1}}{H_{N_2}} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}}}$$

Hence the normal components of H is not continuous at the boundary. The field strengths in two media are inversely proportional to their relative permeabilities.

<u>Boundary conditions for tangential component</u>

According to Ampere's circuital law

$$\oint \bar{H} \cdot d\bar{L} = I$$

Consider a rectangular loop in clockwise direction as a-b-c-d-a. This closed path is placed in a plane normal to the boundary surface

$$\oint \bar{H} \cdot d\bar{L} = \int_a^b \bar{H} \cdot d\bar{L} + \int_b^1 \bar{H} \cdot d\bar{L} + \int_1^c \bar{H} \cdot d\bar{L} + \int_c^d \bar{H} \cdot d\bar{L} + \int_d^2 \bar{H} \cdot d\bar{L} + \int_2^a \bar{H} \cdot d\bar{L} = I$$

T he sides a-b and c-d are parallel to the tangential direction to the surface. While the other two sides are normal to the surface at the boundary

Let the elementary height and elementary width be Δh and Δw

The normal and tangential component in medium1 and in medium2arein opposite direction

The above equation are written as

To get conditions at boundary, $\Delta h \to 0$. Thus,

$$K \bullet dw = H_{tan1}(\Delta w) - H_{tan2}(\Delta w)$$

$$H_{tan1} - H_{tan2} = K$$

In vector form

$$\boxed{\bar{H}_{tan1} - \bar{H}_{tan2} = \bar{a}_{N12} \times \bar{K}}$$

aN12 – unit vector
the above equ in terms of B

$$\frac{B_{tan1}}{\mu_1} - \frac{B_{tan2}}{\mu_2} = K$$

Special case
The boundary is free of current
So K = 0

$$H_{tan1} - H_{tan2} = 0$$

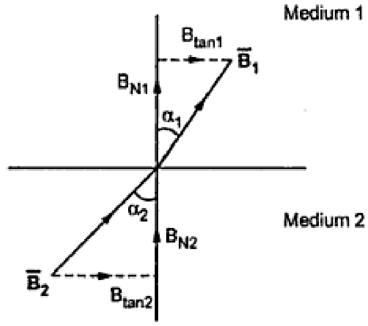
$$H_{tan1} = H_{tan2}$$

Similarly

$$\frac{B_{tan1}}{\mu_1} = \frac{B_{tan2}}{\mu_2}$$

$$\boxed{\frac{B_{tan1}}{B_{tan2}} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$

Condition
From the above equ the tangential components of H are continuous while the tangential components of B are discontinuous at the boundary



In medium 1
Tan $\alpha 1$ = B tan1 / BN1
In medium 2
Tan $\alpha 2$ = B tan2 / BN2
Diving the above equ
Tan$\alpha 1$ / tan $\alpha 2$ = Btan1BN1 .BN2 / B tan2
W.K.T
BN1 = BN2

Tan$\alpha 1$ / tan $\alpha 2$ = Btan1 / B tan2 = μr1 /μ2

UNIT V

19.(a) State & prove Poynting Throrem

Poynting's theorem states that **the rate of energy transfer per unit volume from a region of space equals the rate of work done on the charge distribution in the region, plus the energy flux leaving that region**.

Proof : The energy density carried by the electromagnetic wave can be calculated using Maxwell's equations

as $\quad \text{div } \vec{D} = 0 \quad \ldots\text{(i)} \quad \text{div } \vec{B} = 0 \ \ldots\text{(ii)} \quad \text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ \ldots\text{(iii)}$

$$\text{and Curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \ldots\text{(iv)}$$

taking scalar product of (iii) with H and (iv) with $\vec{E}$

i.e. $$\vec{H}\text{ curl } \vec{E} = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} \qquad \ldots\text{(v)}$$

and $$\vec{E}.\text{ curl } \vec{H} = \vec{E}.\vec{J} + \vec{E}.\frac{\partial \vec{D}}{\partial t} \qquad \ldots\text{(vi)}$$

doing (vi) – (v) i.e. $$\vec{H}.\text{curl } \vec{E} - \vec{E}.\text{ curl } \vec{H} = -\vec{H}.\frac{\partial \vec{B}}{\partial t} - \vec{E}.\vec{J} - \vec{E}.\frac{\partial \vec{D}}{\partial t}$$
$$= -\left[\vec{H}.\frac{\partial \vec{B}}{\partial t} + \vec{E}.\frac{\partial \vec{D}}{\partial t}\right] - \vec{E}.\vec{J}$$

as $$\text{div}\left(\vec{A}\times\vec{B}\right) = \vec{B}.\text{ curl } \vec{A} - \vec{A}\text{ curl B}$$

so $$\text{div}\left(\vec{E}\times\vec{H}\right) = -\left[\vec{H}.\frac{\partial \vec{B}}{\partial t} + \vec{E}.\frac{\partial \vec{D}}{\partial t}\right] - \vec{E}.\vec{J} \qquad \ldots\text{(vii)}$$

But $$\vec{B} = \mu\vec{H} \quad \text{and} \quad \vec{D} = \varepsilon\vec{E}$$

so $$\vec{H}.\frac{\partial \vec{B}}{\partial t} = \vec{H}.\frac{\partial}{\partial t}\left(\mu\vec{H}\right) = \frac{1}{2}\mu\frac{\partial}{\partial t}\left(H^2\right)$$
$$= \frac{\partial}{\partial t}\left[\frac{1}{2}\vec{H}.\vec{B}\right]$$

and $$\vec{E}.\frac{\partial \vec{D}}{\partial t} = \vec{E}.\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right) = \frac{1}{2}\varepsilon\frac{\partial}{\partial t}(E)^2 = \frac{\partial}{\partial t}\left[\frac{1}{2}\vec{E}.\vec{D}\right]$$

so from equation (vii) $$\text{div}\left(\vec{E}\times\vec{H}\right) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\vec{H}.\vec{B} + \vec{E}.\vec{D}\right)\right] - \vec{E}.\vec{J}$$

or $$\vec{E}.\vec{J} = -\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\vec{H}.\vec{B} + \vec{E}.\vec{D}\right)\right] - \text{div}\left(\vec{E}\times\vec{H}\right) \qquad \ldots\text{(viii)}$$

Integrating equation (viii) over a volume V enclosed by a surface S

$$\int_V \vec{E}.\vec{J}\,dV = -\int_V \left|\frac{\partial}{\partial t}\left\{\frac{1}{2}\left(\vec{H}.\vec{B} + \vec{E}.\vec{D}\right)\right\}\right| dV - \int_V \text{div}\left(\vec{E}\times\vec{H}\right)dV$$

or $$\int_V \bar{E}.\bar{J}\, dV = -\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] dV - \int_S \left(\bar{E}\times\bar{H}\right).ds$$

as $\bar{B} = \mu\bar{H}$, $\bar{D} = \varepsilon\bar{E}$ and $\int_V \mathrm{div}\left(\bar{E}\times\bar{H}\right)dV = \int_S \left(\bar{E}\times\bar{H}\right).ds$

or $$\int_V \left(\bar{E}\,.\,\bar{J}\right)dV = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right]dV - \int_S \left(\bar{E}\times\bar{H}\right)ds$$

or $$\int_s \left(\bar{E}\times\bar{H}\right).ds = -\int_V \frac{\partial U_{em}}{\partial t}\,dV - \int_V \left(\bar{E}\,.\,\bar{J}\right)dV$$

or $$\boxed{\int_s \bar{P}.ds = -\int_V \frac{\partial U_{em}}{\partial t}\,dV - \int_V \left(\bar{E}\,.\,\bar{J}\right)dV} \qquad \left(\text{as } \bar{P} = \bar{E}\times\bar{H}\right) \quad ...(ix)$$

i.e. Total power leaving the volume = rate of decrease of stored e.m. energy - ohmic power dissipated due to charge motion

This equation (ix) represents the poynting theorem according to which the net power flowing out of a given volume is equal to the rate of decrease of stored electromagnetic energy in that volume minus the conduction losses.

(b) Describe the Poynting Vector , Pag & Instantaneous Power.
For Answer Refer 19(a)

20. In a material for which σ=5S/m & ε_r= 1, the electric field intensity is E 250 sin 10^{10}tv/m. Estimate the conduction & displacement current densities, & the frequency at which both have equal magnitudes

Given that,

$$\sigma = 5\ S/m$$

Electric field intensity is

$$E = 250\sin 10^{10}t$$

We need to calculate the conductor current
Using formula of conductor current

$$I_c = \sigma E$$

Put the value into the formula

$$I_c = 5\times 250\sin(10^{10}t)$$

$$I_c = 1250\sin(10^{10}t)\ C/m^2$$

We need to calculate the displacement current densities
Using formula of displacement current density

$$I_d = \epsilon_0\frac{dE}{dt}$$

Put the value into the formula

$$I_d = 8.854\times 10^{-12}\times 250\times 10^{10}\cos(10^{10}t)$$

$$I_d = 22.14\cos(10^{10}t)\ C/m^2$$

The frequency at which they have equal magnitude.

$I_c = I_d$

Then, $\sigma = \omega\epsilon_0$

We need to calculate the frequency at which they have equal magnitude

Using formula of frequency

$$\omega = \frac{\sigma}{\epsilon_0}$$

Put the value into the formula

$$\omega = \frac{5}{8.854 \times 10^{-12}}$$

$$\omega = 5.64 \times 10^{11} /sec$$

$$\omega = 89.8\ GHz$$

Hence, The conductor current is $1250 \sin(10^{10}t)\ C/m^2$

The displacement current density is $22.14 \cos(10^{10}t)\ C/m^2$

The frequency is 89.8 GHz.